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MF aerials: the determination of ground-wave c.m.f. and mean ground conductivity from radial field measurements by an optimisation technique

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Summary

A computing method for obtaining c.m.f. and average ground conductivity values from sets of ground-wave measurements taken on radials from an m.f. aerial should be valuable in assessing the performance of new arrays in the future. A suitable optimisation technique which can be employed for this purpose and could be realised as a computer program is described.

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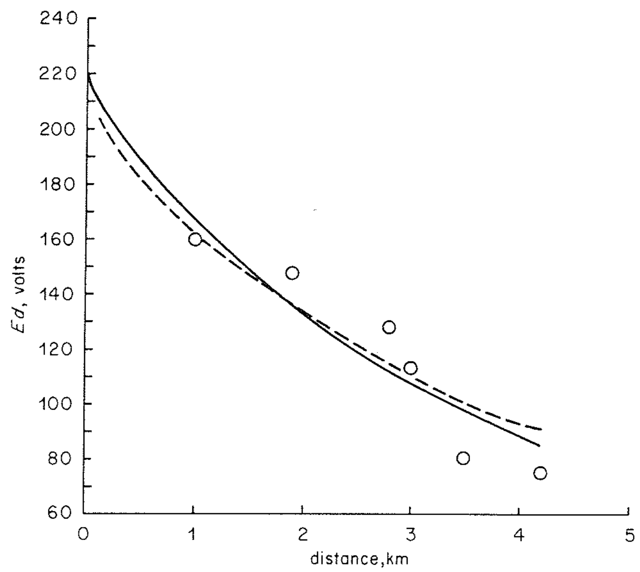
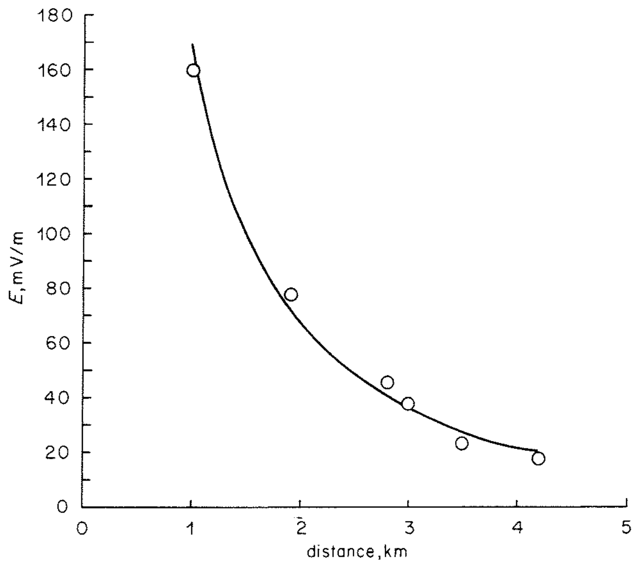
1. Introduction

A rule-of-thumb method has been in general use for estimating the ground-wave c.m.f.* and mean ground conductivity from sets of radial field measurements within a distance of about 5 km from an m.f. aerial. If E is the field measured at distance d, the method is to plot log (Ed) against d on graph paper and to obtain the required results from the best-fit straight line drawn through the plotted points. This procedure can lead to very poor estimates of c.m.f., especially when the effective ground conductivity is low and the frequency is at the top of the band. The purpose of this report is to describe an improved method suitable for the assessment of aerial performance. It uses a non-iterative optimisation technique which can readily be realised as a computer program. This method can use any known theoretical attanuation function and optimises a scaling factor (equivalent to c.m.f.) and the ground conductivity value to give the minimum value of accumulated squared difference from the measured values of field strength. The final values of these quantities are obtained by interpolation.

It is assumed that fields will be measured over the ground (not at sea) and therefore it is possible to limit the expected values of conductivity and dielectric constant.



(*a*)



(*b*)

Fig. 1 - Theoretical curve fitting to measured results

(*a*) Ed (Volts) as a function of distance

———— Approximate formula - - - - - - More exact formula

(*b*) E (mV/m^{-1}) as a function of distance

———— Approximate formula

2. The attenuation function

The ground-wave field strength for short distances (but greater than about 5λ) can be expressed approximately as

$$E = \frac{2E_o}{d}\, \mathrm{A}(p) \qquad (Vm^{-1}) \qquad (1)$$

where A(p), the attenuation function,

$$= \frac{2 + 0{\cdot}3p}{2 + p(1 + 0{\cdot}6p)} \quad - \sqrt{\frac{p}{2.}}\; \mathrm{e}^{-5p/8} \,.\, \sin b$$

In this approximation p is a numerical distance and b is a parameter. Both are defined below. This empirical function, due to Norton[1,2] gives a good approximation to a more exact function as can be seen by reference to Fig. 1(*a*). A more exact expression derived from the theory of Sommerfield is

$$\mathrm{A}(s) = |1 + \mathrm{j}\sqrt{\pi s}\,.\,\mathrm{e}^{-s}\,.\,\mathrm{erfc}(-\mathrm{j}\sqrt{s})|$$

where erfc(z) is the complex error function complement which is tabulated,[3] and $S = pe^{jb}$.

* c.m.f. = cymo-motive force. For the purposes of this report it can be identified with the constant C in Equation (3).

We see that the product Ed is equal to $2E_0$ at $p = 0$, ($d = 0$). $2E_0$ can therefore be identified with the aerial c.m.f. (Volts). Given the ground conductivity σ (milli Siemen m^{-1}), permittivity ϵ (farad m^{-1}) and the frequency f (Hz) the following quantities are required:

$$x = (1{\cdot}8 \times 10^7)\sigma/f \quad (\text{farad } m^{-1})$$

$$\tan b = \frac{(\epsilon + \epsilon_0)}{\epsilon_0 x}$$

$$p = \frac{\pi}{x\lambda}.\cos b\,.\,d. = kd$$

The above formulae refer to vertical polarisation only. Horizontal polarisation is not covered in this report although a similar set of procedures can be devised for it.

3. Ground permittivity

The permittivity of different types of ground is, of course, not a well defined function of conductivity. In temperate climates, however, there is a tendency for permittivity to increase with conductivity which is probably due to increasing water content in the soil. In cities and built-up areas 'real' conductivities and permittivities must be replaced by 'effective' values both of which are low. Rather than use a fixed typical or mean value of permittivity (as is done in CCIR Recommendation 368-1 for example), improved accuracy can be secured by expressing the relative permittivity as a single valued function of conductivity. Thus

$$\frac{\epsilon}{\epsilon_0} = 20\,(1 - e^{-0{\cdot}23\sigma}) \qquad (2)$$

$$\sigma = \text{conductivity in mS, m}^{-1}$$

This formula will not necessarily give the correct value of permittivity in a place of a fixed value but its justification is that on the average smaller errors will be incurred if it is adopted. Fig. 2 shows a plot of this function together with some typical examples of points corresponding to various types of terrain.

4. Measured Ed values

Restating the basic formula in a slightly different form:

$$V = Ed = CA\,(kd) \quad (\text{Volts}) \qquad (3)$$

It will be assumed that a set of measured values of Ed is available which has been taken on a radial from a vertical transmitting aerial within a range of approximately 5 to 200 wavelengths. The given data are therefore a set of points (d_r, V_r) r = 1, 2 . . . n at the frequency of the test transmission. Owing to practical measuring difficulties the number of measurement points may be limited to about a dozen and often less.

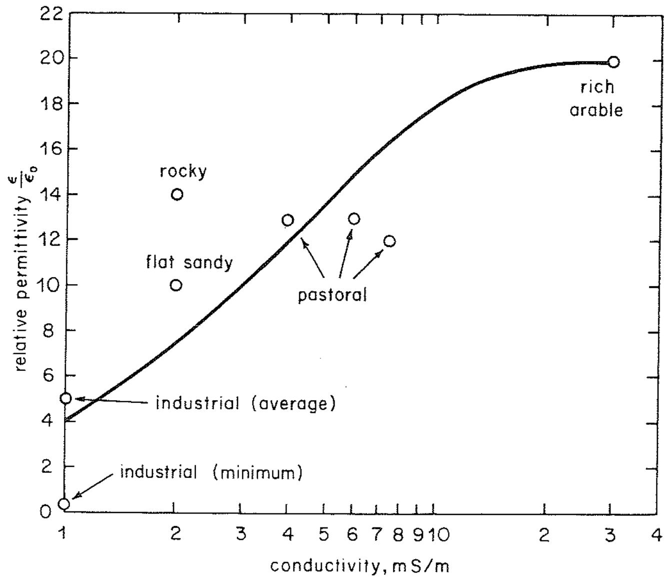


Fig. 2 - Assumed relationship between conductivity and relative permittivity for overland paths

5. Optimisation method

5.1. Step 1

Chose $\{\sigma_s\}$, a set of values of conductivity in the range $0{\cdot}1 \leqslant \sigma \leqslant 20$, say. It is convenient, as far as possible, to have roughly equal increments between the chosen values of σ and a set of about fifteen to twenty is sufficient. Calculate corresponding values of k giving the set $\{k_s\}$. From these values a set of values of the attenuation function can be found from the formulae in Section 2.

$$\{A_{sr}\} \equiv \{A(k_s d_r)\}$$

These values and the points (d_r, V_r) form the arrays on which the optimisation procedure is based.

5.2. Step 2

$\{C_s\}$ is next derived where

$$C_s = \sum_{r=1}^{r=n} A_{sr} V_r \Big/ \sum_{r=1}^{r=n} (A_{sr})^2$$

This process scales the theoretical V-curves for each of the conductivity values by adjusting the multiplying constant C in expression (3) to minimise the sum of squared differences.

5.3. Step 3

This step involves the calculation of the sum of squared differences between measured and calculated values of V.

Thus

$$\delta_s^2 = \sum_{r=1}^{r=n} [V_r - C_s A_{sr}]^2$$

giving $\{\delta_s^2\}$.

6. Functional dependence on conductivity

The multiplier C in Equation (3) is to be thought of as equivalent to the c.m.f. of the aerial system which is being measured. In reality this multiplier has a fixed value determined by the aerial system but in the optimisation process its value changes as different values of conductivity are assumed. In the present context it may therefore be considered as dependent upon the conductivity. It follows that both the multiplier C and the squared difference variable are functions of the conductivity only which means that a one-dimensional interpolation may be used to obtain the best estimate of conductivity and c.m.f. as explained in the next section.

7. Interpolation

The value of conductivity which corresponds to the minimum of δ^2 can be found to good accuracy by means of parabolic interpolation. The interpolation involves the point (δ_l^2 σ_l) giving the minimal calculated value and the neighbouring points on either side. It can be shown that the interpolated value of σ will be an accurate estimate of that value for which δ^2 is a minimum. Having found this optimal value of σ, the best estimate for C (the c.m.f.) may be obtained by linear interpolation or by applying Step 2 (Section 5.2) for this single value of conductivity.

7.1. Step 4

Min $\{\delta_s^2\}$ is found and, if this is a single member of the set, δ_l say, the points $(\delta_{l-1}^2, \sigma_{l-1})$ (δ_l^2, σ_l) and $(\delta_{l+1}^2, \sigma_{l+1})$ are noted and interpolation carried out. The optimal value of σ is given by:

$$2\sigma = \frac{(\sigma_l^2 - \sigma_{l+1}^2)(\delta_{l-1}^2 - \delta_l^2) - (\sigma_{l-1}^2 - \sigma_l^2)(\delta_l^2 - \delta_{l+1}^2)}{(\sigma_l - \sigma_{l+1})(\delta_{l-1}^2 - \delta_l^2) - (\sigma_{l-1} - \sigma_l)(\delta_l^2 - \delta_{l+1}^2)}$$

If there are two sequential members of $\{\delta_s^2\}$ having the same minimal value* either may be chosen as δ_l^2. The likelihood of more than two sequential members having the same value is negligible. If the minimal value belongs to either end member (but not both) it can be taken as the actual minimum. In this event, however, note should be made of the fact that the range of conductivity values may be inadequate. If nominally equal minimal values are possessed by separated members it should be inferred that the measured results are too erratic for smoothing to be applied.

* The possibility of this occurrence will clearly depend on the number of significant figures which are available.

7.2. Step 5

Given the optimal value of σ, obtain the optimal value of C by linear interpolation or by the application of Step 2 for the single optimal value of σ.

8. Example

Fig. 1 shows the result of applying the process to six radial field strength measurements taken from a service broadcast aerial operating at a frequency of 1457 kHz. From the measurements the aerial c.m.f. (in the radial direction) is found to be 221·9 volts and the effective ground conductivity to be 3·07 m$S\ m^{-1}$. This is a fairly low value of conductivity but the path of the radial is known to pass over undulating ground which is well covered with banks of trees and buildings and this no doubt accounts for this fact at the relatively high frequency.

The dotted curve in Fig. 1(*a*) shows a plot of the more exact function mentioned in Section 2 derived from curves given in Reference 1.

9. C.M.F. ground-wave patterns of aerials

One of the principal applications for the process described in the preceding sections is the calculation of c.m.f. ground-wave patterns of vertical m.f. aerials from a series of measurements. This application will be particularly important when new directional arrays are being set to work and the amplitude and phases of their drive currents are being critically adjusted. The advantage of the process is that it minimises ground attenuation effects in the estimation of aerial performance.

10. Ground conductivity

In the author's opinion calculation of m.f. ground-wave services often assume values of effective ground conductivity which are too high when the operating frequency is at the top of the m.f. band (above 1 MHz). Ground undulations, trees, building and other irregularities cause increased attenuation whereas predictions are often based on surface stratum values of conductivity and measurements made at low frequencies. Application of the optimisation process to measured fields from aerials could lead to improved estimates for the average conductivity to be assumed in future work.

11. Conclusions

An explicit non-iterative optimisation process has been described for obtaining the ground-wave c.m.f. values and local effective ground conductivity of vertical m.f. aerials from measurements. The accuracy of the method as a curve fitting process is believed to be good and its application in assessing the performance of new m.f. arrays in the future will, it is hoped, be valuable.

12. References

1. NORTON, R.A. 1936. The propagation of radio waves over the surface of the earth and in the upper atmosphere (Part 1). *Proc. IRE*, 1936, **24**, 10, pp. 1367 – 1387.

2. NORTON, R.A. 1941. The calculation of ground-wave field intensity over a finitely conducting spherical earth. *Proc. IRE*, 1941, **29**, 12, pp. 623 – 639.

3. ABRAMOWITZ, M. *and* STEGAN, I.A. 1964. Handbook of mathematical functions. Dover, New York, 1964.

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